Predicting the Performance of a Human Powered Vehicle

ENG1102 at Michigan Technological University

Newton's laws apply:

$$\Sigma F = ma$$

If the drive force is greater than all the resistance forces, the vehicle will accelerate. If all the forces acting on the vehicle balance (add up to 0), the vehicle will stay at a constant speed.

Forces:

F_P = Propulsion Force (usually where the rubber meets the road)
F_A = Drag Force (air resistance)
F_S = Slope Resistance (force to hold vehicle and rider on a hill)
F_R = Rolling Resistance (friction force due to tire deformation)
F_B = Bump Resistance (average force due to lost energy over bumps)

Masses:

m_V = Mass of the Vehicle
m_R = Mass of the Rider(s)
m_C = Mass of the Cargo

Propulsion Force:

The “engine” in an HPV is the rider(s) whose output is usually measured in terms of power (watts or lb-ft/s). We also know that:

Power = Force x Velocity

So, if we convert the rider’s output from watts to lb-ft/sec (or N-m/s) and use the vehicle’s velocity, we can calculate the propulsion force:

$F_P = P / V$ where, P = rider’s output power
V = vehicle’s velocity

Riders vary tremendously in their output; i.e., from 50 watts for a casual ride in the park up to 2000+ watts for a peak athlete’s sprint. You can estimate a rider’s output based on age, weight, physical condition, sense of urgency, etc. See the attachments for some specific power data.

Unfortunately, all the power does not get from the rider to the ground because the drive system (bearings, chain, sprockets, etc) soak up some of the power. We usually lump these mechanisms together into an overall “mechanical efficiency”, $\varepsilon_{drive.}$ The typical bicycle drive train (in good condition) has an efficiency in the range of 85 – 95%. So, what we end up with is:

$$\mathbf{F_P = P * \varepsilon_{drive} / V}$$

Drag Force:

Aerodynamic drag is generally described and shown in attachments. Once you know the vehicle’s frontal area (size), **A**, and drag coefficient (shape) $\mathbf{C_D}$, you can get a good estimate from:

$$\mathbf{F_A = \frac{1}{2} \rho A C_D V_A{}^\wedge 2}$$

where, ρ = mass density of air = 1.29 kg/m^3
A = frontal area
$\mathbf{V_A}$ = air velocity
$\mathbf{C_D}$ = drag coefficient

Note that air velocity is usually NOT the same as vehicle velocity --- you may need to factor in wind speed and direction.

Slope Resistance:

If we define F_S as the force to balance the weight, W, of the vehicle tangent to the road grade, then:

$$\mathbf{F_S = m * g * \%Grade}$$

where, % Grade = rise / distance traveled
m = total mass of vehicle and rider(s)
g = acceleration due to gravity

The power, P_S, used to move up the grade (change in potential energy) is:

$$P_S = F_S * V$$

where, V = vehicle velocity

Rolling Resistance:

The deformation of the tire absorbs some energy when it rolls. Hence, a force is needed to keep it rolling. This is usually simplified to an empirical relationship:

$$\mathbf{F_R = C_R * m * g}$$

where, $\mathbf{C_R}$ = coefficient of rolling resistance

Except for heavy vehicles at low power, the rolling resistance is very small compared to other losses. Typical values for $\mathbf{C_R}$ are shown in the attachments.

Bump Resistance:

It is obvious to most bike riders that bumps absorb some of your momentum. However, this is a difficult effect to model and not many studies have provided clear data. Typically, the resistance is measured in terms of power. Data gathered by Pradko and Lee in 1966 found the highest power loss to be 2000 W and the lowest to be 2.7 W (*Bicycle Science, Wilson & Papadopoulos, MIT press, 2004*). To get an estimate of the associated bump resistance force, F_B, we can re-arrange the power equation:

$$\mathbf{F_B = P_B / V}$$

where, P_B = bump power loss (lb-ft/s) or (N-m/s)
Estimates: for smooth, new blacktop, P_B = 5 W
for a rough, gravel road, P_B = 100 W

For more accurate estimates, see attached graph of wattage as a function of bump frequency and displacement. It should also be noted that the effect of road roughness can be reduced with vehicle suspension (or rider suspension) so that the vibrations created by the road are isolated from the major mass of the vehicle.

<u>Notes on Units:</u>

Engineers in the US get stuck using both US and metric systems of measure:

For US system---- Force (lb) = mass (slugs) x acceleration (ft/sec^2)

Mass (slugs) = weight (lb) / 32.2 (ft/sec^2)

Power: 1 watt = 0.73757 ft*lb/s

For SI (metric) --- Force (N) = mass (kg) x acceleration (m/s^2)

Mass (kg) = weight (N) / 9.8 (m/s^2)

Power (N-m/s) = Power (watts)

Predicting Vehicle Performance

Once the forces acting on the vehicle are known, we can calculate the vehicle's acceleration, a:

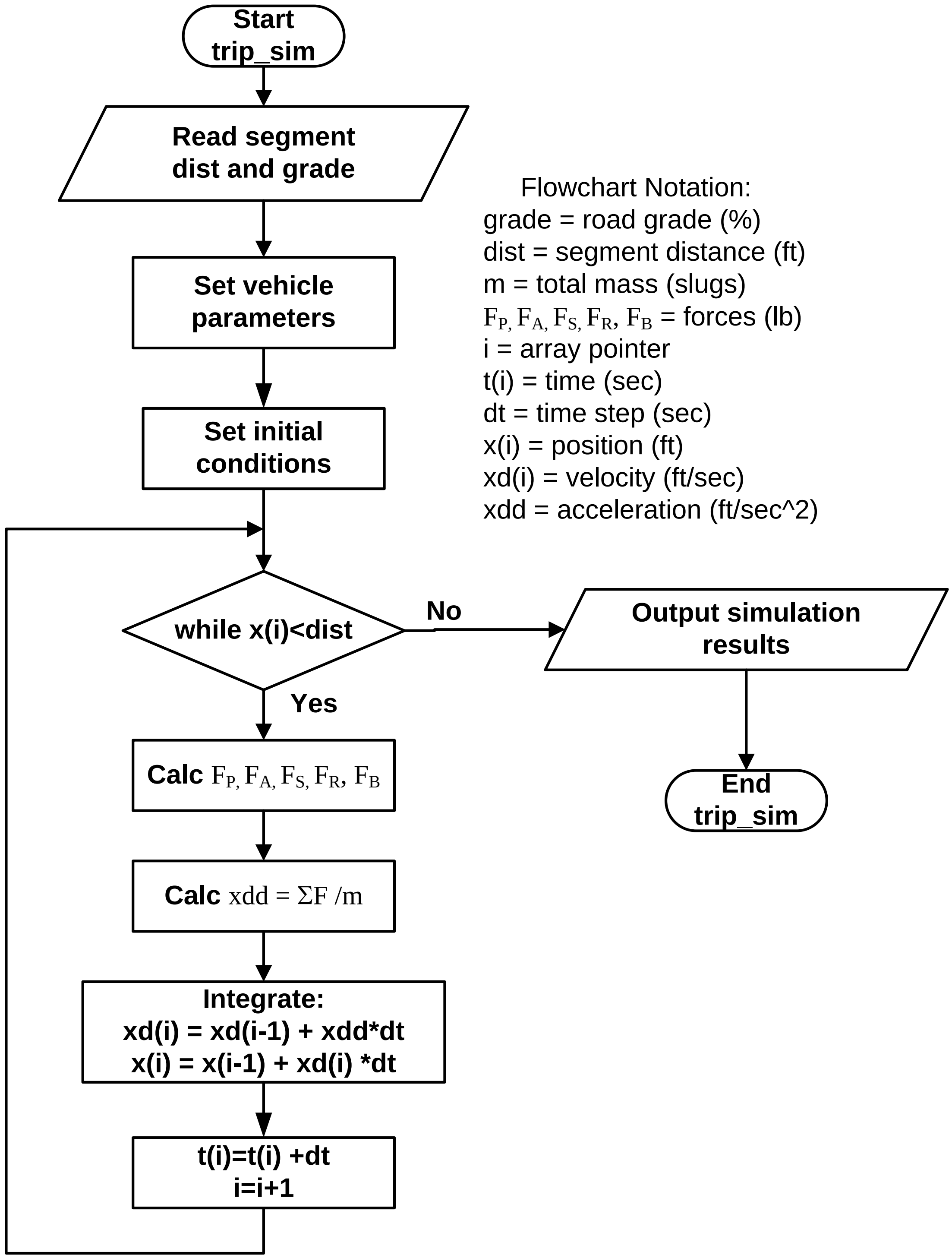
a(t) = ΣF/m where, everything except mass is changing with respect to time.

So, if we integrate acceleration with respect to time, we create an estimate of vehicle velocity. And, if we integrate velocity, we create an estimate of the vehicle's position.

$$V(t) = \int_0^t a(t) \bullet dt$$

$$x(t) = \int_0^t V(t) \bullet dt$$

Hence, we can simulate the vehicle's performance using numerical integration as shown in the following (approximate) flowchart.



Flowchart Notation:

grade = road grade (%)
dist = segment distance (ft)
m = total mass (slugs)
F_P, F_A, F_S, F_R, F_B = forces (lb)
i = array pointer
t(i) = time (sec)
dt = time step (sec)
x(i) = position (ft)
xd(i) = velocity (ft/sec)
xdd = acceleration (ft/sec^2)

Attachment

The following is a sampling of data taken from *Bicycling Science by David Wilson and Jim Papadopoulos, MIT Press, 2004.*

A. Human Power Curves (p 44)

B. Model of Leg Mechanics (p 50)

C. Power vs Age (p 64)

D. Power and Breathing Rates for cyclists (p76)

E. Typical HPV parameters (p139, 140)

F. World Records (p152)

G. Energy Costs for different modes of transport (p161)

H. Airflow and Drag (p174,5))

I. Example Bicycle Drag Coeffients (p188)

J. Drag Coefficients (p191)

K. Tandem (Drafting) Bodies (p198,199)

L. Rolling Resistance (p226,227,230)

M. HPV Sprint Plot (p407)